Introduction to Baravelle Spiral Interdisciplinary Unit

Linda Stowe
Perpich Center for Arts Education

Definition of Baravelle Spiral

A Baravelle Spiral is not actually a curved design. It is made entirely of straight lines and only appears to look curved. The spiral effect comes from the way the structure is drawn. To draw a Baravelle Spiral, begin with any regular polygon and find the midpoints of each side of that polygon. Connect all those midpoints with straight lines, creating a polygon within a polygon. Find the midpoints of each of the sides of this new polygon and connect them with straight lines. Continue doing this as long as it is humanly possible to do so or until you run out of patience. Next you need to shade some of the triangles. To create one spiral, begin by shading any one of the outer triangles (also the largest of them). Move in to the next level of triangles, either in a clockwise direction or a counter-clockwise direction and shade the next size triangle that touches the first one shaded. Continue this to the center of the polygon. Several examples appear below.

Figure 1 uses a triangle with the triangles shaded in a clockwise direction.
Figure 2 uses a pentagon with the triangles shaded in a clockwise direction.

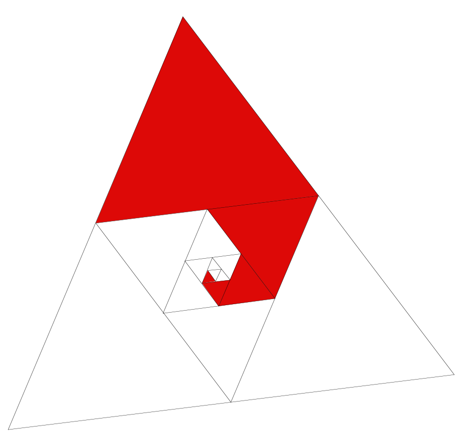

Figure 1

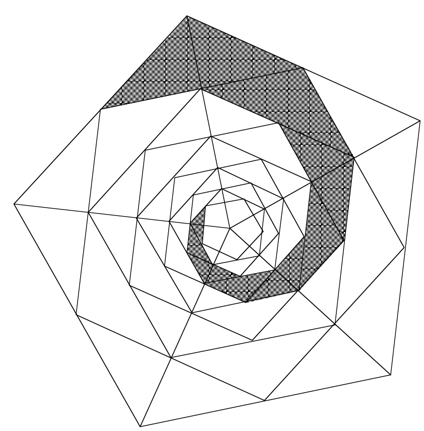

Figure 2

Figure 3 is made from hexagons and the spiral is drawn counterclockwise.

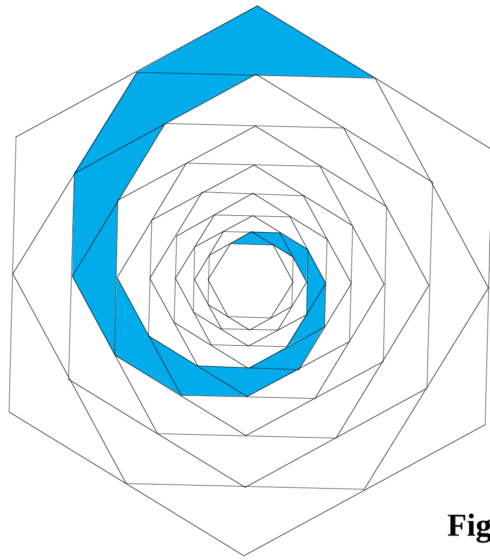

Figure 3

Multiple spirals can be shaded in a given shape. A hexagon, for example, could have six Baravelle Spirals going in a clockwise direction and six going in a counter-clockwise direction.

What Do Baravelle Spirals Have to Do with Mathematics Curriculum?

These spirals are closely related to the concept of geometric sequences and series. The areas of the triangles of any one Baravelle Spiral form a geometric sequence, starting with the area of the largest triangle, followed by the area of the next largest triangle, and so on. If the triangle is constructed quite accurately the base and height of a triangle can be measured and the areas determined with these measurements. In this case, the ratio between the terms of the sequence will not be exact but should be fairly close. The accuracy depends on how well the drawing is done and how well the measurements are taken. For students who know trigonometry, these measurements can be determined very accurately and thus the ratios that result are very accurate and it can easily be determined that they indeed are the same ratio.

On a deeper level, the concept of the infinite geometric series can be related to the Baravalle Spiral. It can be argued that a Baravelle Spiral consists of an infinite number of triangles. An individual obviously has to stop drawing triangles when they get very small but in the mind the triangles and thus the spiral continue indefinitely. Thus, it is accurate to say that the area of one of these spirals can be computed by finding the sum of an infinite number of terms forming the geometric sequence. There exists a formula for this kind of situation when the constant ratio is a positive number less than one. No matter what regular polygon you start with when constructing a Baravelle Spiral, the common ratio is always positive and less than 1. Thus, the sum can be determined.

To reinforce that this is well and good, it can further be observed that n spirals of exactly the same size will fit into an n-gon. So by computing the area of the entire n-gon and dividing by n will give the area of one spiral. This value will always agree with the area computed using the sum of the infinite geometric series formula—giving the student a concrete example that the formula is valid!

Lesson Plans to be Used in Mathematics Classes

Notes from the teacher: I teach the mathematics concepts listed above in Advanced Algebra. They are also often taught in Pre-Calculus or other classes. When we finish the formulas for the sequences and series, I continue the unit with the following steps. I teach in blocks of 90 minutes per day so these can be adjusted accordingly.

Day 1:

Introduce the idea of a Baravelle Spiral and show on the overhead what several look like. I also have previous drawings done by students hanging in the classroom and I point these out.

Have students create a Baravelle Spiral using graph paper and using a square as the polygon. I tell everyone to draw a square using a multiple of 4 as the length of the side of the square. Squares are easiest to do and using graph paper makes it even easier. I go through the steps on the overhead of creating a Baravelle Spiral. Each student is asked to shade in one spiral on his/her design.

I hand out the worksheet entitled Baravelle Spirals and have the students work on these problems. (I always review how to find the area of a triangle on the graph paper—using one square on the paper as having an area of one.) After some work time, I summarize by having students share what results they got. If students aren't finished they can complete this at home.

Day 2:

Explain to the students the project they are to be working on (see Day 3—7 for possibilities). To begin with each student will be creating a Baravelle Spiral on a piece of white unlined paper. I usually have posterboard for them. Allow students time to choose an n-gon of their choice. I do not allow squares for this project. I review techniques for creating a regular polygon on unlined paper. Then the class is given time to construct their Baravelle Spiral. If students do not have enough time to get their polygons drawn they are assigned it as homework.

Day 3--7: We work on the art integration. This can take several forms. I am fortunate to have a visual art teacher come in to help with this. The visual art teacher does work on the first day regarding color mixing and the class practices mixing paints and practices various application methods, keeping in the back of their mind, that they are to do decide what he/she will do with their spiral in their own project. We have some kind of art project that the students incorporate their Baravelle Spirals into. Several possibilities have been tried over the years. One project was to make a floor rug, giving each student a square of fabric to paint their design onto and then they were sewn together and shellacked. Another time, students painted their design on the walls of the hallway near the classroom. On occasions where there wasn't as much time or the classes were larger, the final project was a paper version with watercolors as the medium. We have had students write artists statements to go along with their work of art and placed these statements near the displayed artwork.

Student Reaction—Benefits

Teaching at an arts school doesn't mean that all students are good at this kind of art or not intimidated by having to draw. However, students really get into this project and do amazingly good work. They really like it and are very proud of their work.

And, they are doing lots of mathematics. I assign a worksheet to compute the area of the Baravelle Spiral that was drawn for the project. On this assignment, students do something similar to the first worksheet computing areas of the triangles and the spiral. The worksheet is called "Baravelle Spiral Project". Since my students have not had trigonometry, I have them measure all dimensions with a ruler.

For the unit test, I include a question or two about the project to test their understanding of the math concepts and how it relates to the Baravelle Spiral.

Resources

Magazine:
Choppin, Jeffery, "Spiral Through Recursion", Mathematics Teacher, October 1994, pp 504–508

Books:
Boles, Martha & Newman, Rochelle, Universal Patterns, Pythagorean Press, 1990

Venters and Ellison, Mathematical Quilts—No Sewing Required, Key Curriculum Press, 2002

Baravelle Spirals

Directions: Using graph paper, make a series of embedded squares, making sure that the side of the first square has an even number of units along one side. After completing the drawing of embedded squares to form a baravelle spiral, shade one spiral with a pencil. Use this spiral to complete the exercises below.

1. Find the areas of the triangles forming the spiral, beginning with the largest triangle. Write the areas of the triangles as a sequence, continuing to the fourth term.

2. Find the sum for an infinite number of terms of the sequence formed in question 1.

3. Find the area of the square you began with (the largest square in the design.)

4. Divide the area of the square (answer to question 3) by 4. How does your answer relate to the answer in question 2? Explain this relationship.

Baravelle Spirals Project

Name_________________________

turn this in with your completed Spiral

1. Choose any one spiral of your polygon. Calculate the areas of the largest four triangles in the spiral you chose. Calculate the areas by measuring each distance to the nearest tenth of a centimeter. Place your measurements are the area in the table below.

	Measure of the base of the triangle (b)	Measure of the height of the triangle (h)	Area of the triangle .5 * b * h
The largest triangle of the spiral			
The second largest triangle of the spiral			
The third largest triangle of the spiral			
The fourth largest triangle of the spiral			

2. Write the areas of the triangles as a sequence, starting with the area of the largest triangle.

3. The sequence is geometric.

a. Calculate the common ratio (r). Round to two decimal places. ____________________________

b. Calculate the sum of the infinite number of terms of the sequence. _________________________

4. Find the area of the largest polygon in your design.

Do this by using the formula:
A = .5 a s n

a is the length of the apothem
s is the length of one side of the polygon
n is the number of sides the polygon has.

Again, measure to the nearest tenth of a centimeter.

a = ______________________ s = ____________________ n = ______

Area of the polygon: _____________________

5. Divide the area of the polygon by the number of spirals that would fit into the largest polygon. ____________________________

**This answer should be the same as the answer to 3b. If not, ask for help.